ON THE STUDY OF THE SUBSTRUCTURE OF GALAXY CLUSTERS: S-TREE TECHNIQUE IN NON-POINT APPROXIMATION

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1 Introduction

The study of substructuring properties of clusters of galaxies is one of the basic problems on the way of understanding of the mechanisms of formation of the large scale structure of the Universe. It is determined by the fact that, the dynamical time scales of clusters of galaxies are comparable with their ages, and hence their hierarchical properties do contain essential primordial information.

A number of statistical methods are developed for the investigation of the substructures of galaxy clusters [10, 6, 2]. Most of the methods are based on the analysis of the positional information, combined with the use of redshift information. For example, wavelets (see e.g.[4]) are usually used to analyze the data on 2D coordinates of galaxies, though the wavelets can be applied for 1D redshift information as well. However such procedure is not self-consistent and can lead to various biases.

From this point of view the S-tree technique developed by Gurzadyan, Harutyunyan and Kocharyan (see[5, 6]) is using in a self-consistent way the positional, kinematical and magnitude information about the system by means of consideration of the phase space of the N-body systems and revealing its typical properties. S-tree method has been applied already for the study of substructure of the Local Group[7], the core of Virgo cluster[11], and Abell clusters of ESO Key Program survey (ENACS)[8].

The existing version of S-tree[6] is based on the 'point' approximation of the N-body system, i.e. it neglects the dimensions of galaxies. However, as distinct to stellar dynamics, when the point approximation is usually quite a fair one, the 'non-point' features of galaxies can be ocassionally important in the study of clusters of galaxies.

Below we will inquire into the development of S-tree technique, in non-point approximation, namely, considering the galaxies as spheres of given

radius, so that the cluster should appear as a set of N spheres.

Our second aim is the description of a refined algorithm of analysis of the numerical output information of S-tree.

2 S-tree technique

First, let us briefly summarize the basics of S-tree technique; for details we refer to the original papers[6, 1] and to earlier references therein.

Consider N-body system as X set. $X = \{x_1, x_2, \ldots, x_N\}$, where $\forall x_i, \quad (i = 1, \ldots, N)$ are given as follows

$L_i(l_i^1, l_i^2, l_i^3)$ $\quad L_i$ is coordinates vector;

$V_i(v_i{}^1, v_i{}^2, v_i{}^3)$ $\quad V_i$ is velocity vector

and M_i are the masses of particles.

Consider the following P function :

$P : X \times X \to R_+$ and $\rho \in R_+$. Now we give the basic definitions.

Definition 1.1

We will say, that $\forall x \in X$ and $\forall y \in X$ are ρ-bound, if:

$$P(x, y) \geq \rho.$$

Definition 1.2

We will say, that $U \subset X (U \neq \emptyset)$ is ρ-bound group, if:

a. $\forall x \in U$ and $\forall y \in X \setminus U \Rightarrow P(x, y) < \rho$

b. $\forall x \in U$ and $\forall y \in U \ \exists x = x_{i_1}, x_{i_2}, \ldots, x_{i_k} = y$,

such $P(x_{il}, x_{il+1}) \geq \rho \quad \forall l = 1, \ldots, k-1$.

Definition 1.3

We will say, that $U_1, \ldots, U_d$ is the distribution of set X by ρ-bound groups, if:

a. $\bigcup_{i=1}^{d} U_i = X$

b. $i \neq j \quad (i = 1, \ldots, d; j = 1, \ldots, d) \Rightarrow U_i \bigcap U_j = \emptyset$

c. $\forall U_d \quad (i = 1, \ldots, d)$ is ρ-bounded group.

Note that any P function can be represented as $A(a_{ij})$ matrix [1].

Thus, we can have a representation of any N-body system via S-tree diagram algorithm for given A matrix and $\forall \rho$. This distribution will satisfy the definitions 1.1,1.2 and 1.3. This algorithm is efficient for any given ρ, so that one will obtain the evolution of dependence of groups on ρ. We will represent the final results in the form of tree-graph (S-tree) or a table. The way one has to choose the final distribution is described below.

3 Distribution via D matrix

Consider again the set X. Via $\bar{M}_i$ let us denote the projection of M_i on l^1l^2 plane. Introduce the following set: $Q = \{Q_1(\bar{M}_1, R_1), \ldots, Q_N(\bar{M}_N, R_N)\}$,

where $R_i \in R_+$, $i = 1, \ldots, N$ and
$Q_i(\bar{M}_i, R_i) = \{(l^1, l^2) | (l^1 - l_i^1)^2 + (l^2 - l_i^2)^2 \leq R_i^2\}$.
Consider $\tilde{P}$ function, as:
$\tilde{P} : X \to Q, \quad \tilde{P}(x_i) = Q_i(\bar{M}_i, R_i)$
For $P_{\tilde{p}} : Q \times Q \to R_+$ and $\forall \rho \in R_+$ we will give definitions, which are equivalent to definitions 1.1-1.3.

Definition 2.1 We will say, that $\forall x_i \in X$ and $\forall x_j \in X$ are ρ-bound, if:

$$P_{\tilde{p}}(Q_i, Q_j) \geq \rho, (i \neq j)$$

.

Definition 2.2 We will say, that $U \subset X (U \neq \emptyset)$ is ρ-bound group, if:
a. $\forall x_i \in U, \forall x_j \in \bar{U} \Rightarrow P_{\tilde{p}}(Q_i, Q_j) < \rho$
b. $\forall x_i \in U, \forall x_j \in U (i \neq j)\ \exists x_i = x_{i_0}, x_{i_1}, \ldots, x_{i_k} = x_j, \forall t \in (0, \ldots k-1)$

$$P_{\tilde{p}}(Q_{i_t}, Q_{i_t+1}) \geq \rho$$

.

Definition 2.3 We will say, that $U_1, \ldots, U_d$ is the distribution of the ρ-bound groups of the set X, if:
a. $\bigcup_{i=1}^{d} U_i = X$
b. $U_i \bigcap U_j = \emptyset$, if $i \neq j, \forall i, j = 1, \ldots, d$
c. $\forall U_i \quad i = 1, \ldots, d$ is ρ-bounded group.
Note, that $P_{\tilde{p}}(Q_i, Q_j) = 0 \quad \forall i$.

Obviously, any $P_{\tilde{p}}$ function can be represented as $D(d_{ij})$ matrix. Now we will show a way of construction of the matrix D.

If $i = j \quad \forall i, j = 1, \ldots, N \quad d_{ij} = 0$.

If $i \neq j$

$$d_{ij} = \begin{cases} 0 & \text{if } \bar{Q}_i \bigcap \bar{Q}_j = \emptyset \\ S_{Q_i \cap Q_j} & \text{otherwise} \end{cases}$$

where $\bar{Q}_i = Q_i \setminus \Gamma_i$, Γ_i-is bound of Q_i and $S_{Q_i \cap Q_j}$ is the area of intersection of Q_i and Q_j. When we know R_i, R_j, $\bar{M}_i$, $\bar{M}_j$ values, it is easy to calculate the area $S_{Q_i \cap Q_j}$.

So, using S-tree diagrams algorithm, we can obtain the distribution by ρ-bounded groups of X set according to defintions 2.1-2.3.

If we introduce also the following function

$$H = P_{\tilde{p}} \circ \hat{P},$$

where $\hat{P} : X \times X \to Q \times Q$.

$\hat{P}(x_i, x_j) = (\tilde{p}(x_i), \tilde{p}(x_j)) = (Q_i, Q_j)$, then we can use only 1.1-1.3 definitions, so long as

$$H : X \times X \to R$$

.

4 Output information algorithm

The results of S-tree analysis can be represented in a form of a table [5, 6]. For optimization and simplification of the search of final distribution of the system, i.e. the search of the necessary floor of the table, one can use the following algorithm.

First, one has to perform a proper transformation of the latter table. This will enable us to obtain the information on the quantitative structure of each distribution in a convenient form. This transition needs $M \times N$ actions, where $(M \times N)$ is (a_{ij}) matrix dimension. We correspond a distribution of U_i numbers to each floor, so that

$$U_i = \sum_{j=1}^{N} a_{ij}, \quad i = 1, \ldots, M.$$

It is easy to see, that the first and the last floors of the matrix (a_{ij}) are

$$U_1 = N$$

$$U_M = \sum_{j=1}^{N} j;$$

In some sense U_i is reflecting the distribution 'density' of the system's subgroups. In order to decrease the time of the run of algorithm it is convenient to stop the investigation of matrix at some U_p, where p is the number of the floor with $U_p \geq S$. One can choose, for example, as x of U_M, so that x will depend on the matrix dimension.

By the next step we construct a chain of auxiliary matrices, in order to obtain the final K matrix reflecting the dynamics of quantitative and qualitative distribution of changes.

Each ith matrix is reflecting the quantitative dynamics of ith subgroup. At the same time, ith matrix can be represented as a function:

$$f_i^*(\rho_t) = n_t^i,$$

where $i = 1, \ldots, N$, $t = 1, \ldots, M$ and n_i^k is the number of ith subgroup at tth floor.

Note, that if we 'paste together' all mentioned auxiliary matrices by 'floors', we will obtain exactly the sought-for K matrix.

Now let us turn to the matrix K. We will consider the behavior of the subgroup, which contains maximal points ($m \geq 2$). We will fix $\Delta\rho_i$ interval, i.e. $\Delta\rho_i$-number of floor, when the group is preserving its main structure or if the group is loosing, for example, the x structure. Note, that x has to be defined for each matrix: if the group is loosing more than x of its numbers, then the process will continue up to the next floor.

Obviously, this process is finite, so far as the number of subgroups c_i is limited:

$$c_1 < c_i < c_M$$

, where $c_1 = 1$ and $c_M = N$, i.e. the first floor contains all particles from a single group (whole system), while on the last floor each particle represents a separate group. This readily follows from the Definitions 1.1-1.3 and the method of choice of ρ_t. Then, we have to choose the maximal $\Delta\rho_i$ and will take the corresponding distribution. As a final distribution we will consider the floor, which results after the destruction of the mentioned distribution.

For the realization of this algorithm we need about $D = M^a \times N^b$ actions [9], where $a, b > 0$.

Let us schematically illustrate this algorithm on a concrete example.

$$\begin{pmatrix} A & A & A & A \\ A & B & A & A \\ A & B & A & C \\ A & B & C & D \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & & 3 & 4 \\ 1 & & 3 & \\ 1 & & & \end{pmatrix} \circ \begin{pmatrix} & & & \\ & 2 & & \\ & 2 & & \\ & 2 & & \end{pmatrix} \circ \begin{pmatrix} & & & \\ & & & 4 \\ & 3 & & \\ & & & \end{pmatrix} \circ \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & 4 \end{pmatrix}$$

$$\mathrm{K}=\begin{pmatrix} 1 & 2 & 3 & 4 & * & & & & & \\ 1 & & 3 & 4 & * & 2 & & & & \\ 1 & & 3 & & * & 2 & * & 4 & & \\ 1 & & & & * & 2 & * & 3 & * & 4 \end{pmatrix}$$

5 Conclusion.

We have proposed some new ways of development of S-tree method in order to take into account the effects which could be important for description of clusters of galaxies. Namely, we considered the possibility of accounting for the non-zero dimensions of the galaxies. The described generalization of S-tree can enable the construction of new type N-body models and can be important especially while studying the denser core regions of galaxy clusters.

We also represented an efficient procedure for the reliable analysis of the output information of S-tree. As numerical experiments, as well as the S-tree runs of the real galaxy clusters indicate, this procedure can become especially useful, when the numbers of galaxies in the studied samples can become about 1000 and more. The obtaining of samples over thousand of galaxies with known redshifts and magnitudes in the vicinity of a given filament, is within the aims of several forthcoming observational programs (for reviews see Durret et al 1994).

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